Insights

FTSE Russell

Top-down or bottom-up?

Balancing exposure and diversification in multi-factor index construction

Executive summary

- For designers of factor indexes there is an inherent trade-off between factor exposure and index concentration.
- Different methods exist to achieve factor exposure in indexes, each with its own merits and drawbacks. In this Insights we compare basket selection approaches with factor tilting, optimization and ad hoc methods.
- In multi-factor indexes, a "top-down" construction approach involves averaging the weights of single factor indexes. By contrast, a "bottom-up" approach involves consideration of all factor characteristics simultaneously.
- We assess these index construction approaches through a variety of metrics: factor exposure, tilt strength, the number of factors targeted and index concentration.
- A bottom-up approach based on tilting typically provides stronger factor exposure and greater diversification than a top-down approach.

For a detailed discussion and further insights into the tilt-tilt approach read our research, "Factor Exposure and Portfolio Concentration."

In recent decades academic researchers have provided theoretical and empirical evidence that risk premia exist for certain equity factors, such as value, momentum, size, low volatility and quality. Market participants continue to adopt factor based investment strategies and providers are launching products linked to factor-based indexes.[1]

[1] Refer to the 2017 FTSE Russell Smart Beta Survey for further information on industry trends; http://www.ftserussell.com/smart-beta-survey.

The rise in popularity of factor investing has seen the focus of debate shift from the existence of factors *per se* to due diligence and an understanding of the key differences between factor strategies. A key question for many investors is how to access factor risk premia and which index construction techniques to use to do this efficiently.

For designers and users of factor indexes there is a trade-off between factor exposure and diversification. In other words, while the factor exposure of an index can be enhanced by increasing the weights to stocks with high factor exposure and lowering the weights of or excluding stocks with low factor exposure, this comes at the expense of index diversification.

In this Insights, we assess the benefits and drawbacks of the various single factor and multi-factor portfolio construction techniques using metrics such as factor exposure, the number of factors targeted, and concentration measures such as Effective N. On the basis of this analysis, we draw conclusions about which approach can deliver factor exposure in the most efficient manner.

Factor index construction methodologies

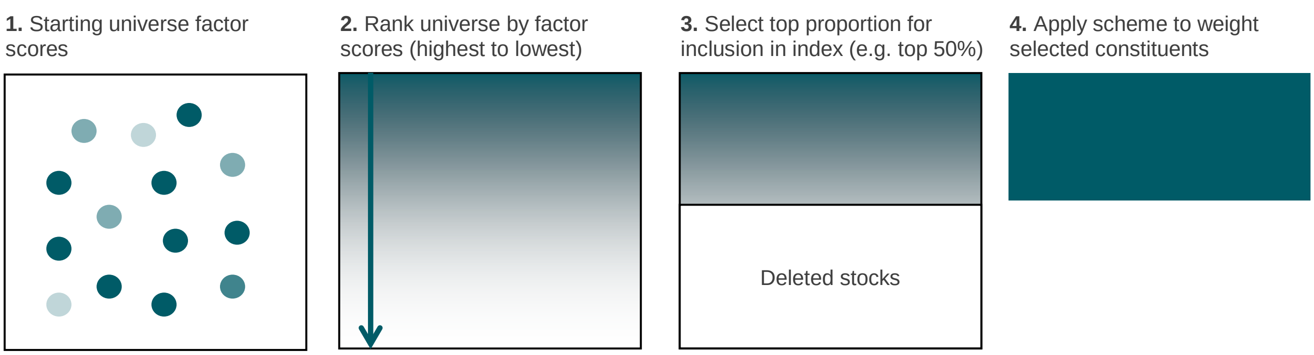
Single factor indexes

Characteristic basket selection

The simplest and most commonly employed factor index construction technique involves a scoring, ranking and exclusion process. A given proportion (e.g., the top 50%) of a stock universe, which has been scored and ranked by a factor value, is chosen to form the characteristic basket.

Once the characteristic basket has been chosen, stocks within the basket may be weighted in different ways. Index weights could be set in proportion to the factor value itself, or according to criteria concerned with specific index objectives, such as capacity (e.g., market capitalization weighting), diversification (e.g., equal weighting) or risk (e.g., risk weighting). In some cases, a combination of these weighting schemes is used.

Exhibit 1. Characteristic basket factor index construction



Source: FTSE Russell.

Advocates of the characteristic basket approach for factor index construction argue that factor exposure can be readily increased by further narrowing the selection universe (for example, setting the cut-off point at the top 30%, rather than the top 50% of the universe, once stocks have been scored and ranked). While this approach undoubtedly increases the index's factor exposure, it comes at the expense of increased concentration.

Factor tilting

An alternative factor index construction approach is factor tilting.[2] The basic idea behind tilting is to start with an initial set of index weights and to reallocate them to increase the index's factor exposure. The starting weights need not be those of the capitalization-weighted index—any portfolio can be tilted towards a single factor or multiple factors in this way.

As in the characteristic basket selection approach, we define a set of factor values for each member of the starting universe to describe its factor characteristics. However, rather than using raw factor scores, which involve different sets of natural units and ranges, in the tilting process, it is convenient to rescale and truncate factor values into standardized factor scores (or "Z-scores"). Z-scores are units of standard deviation which express a stock's factor characteristics in relation to the average of all the other stocks in the universe.

FTSE Russell's chosen factor index construction approach is to use Z-scores in a range from minus 3 to plus 3. Plus 3 (or minus 3) indicates the highest (or lowest) scoring stock in the universe according to the chosen factor, and the average is represented by a Z-score of 0.

In order to effectively tilt portfolios (towards or away from target factors), Z-scores need to be converted (or "mapped") to positive numbers. The choice of the mapping approach used to achieve this (a mathematical concept sometimes called a "functional form" or "tilting function") is important as it determines many of the properties, and specifically the level of factor exposure, of a factor index.

The tilting function should satisfy two important characteristics: it should be non-decreasing (i.e. stocks with higher factor values should not receive smaller weights than stocks with smaller factor values); and it should limit the overweighting or underweighting of stocks with extreme Z-scores.

These conditions imply an "S" shaped curve, such as the Cumulative Normal function shown in Exhibit 2, for mapping Z-scores (shown on the X-axis) to the tilt coefficient (shown on the Y-axis, "S-score"). FTSE Russell uses this tilting function in its factor index construction.

[2] Asness, C. "The Value of Fundamental Indexing", *Institutional Investor*, 94-99, October 2006, Bender, J. and T. Wang "Tilted and Anti-Tilted Portfolios: A Coherent Framework for Advanced Beta Construction", *The Journal of Index Investing*, 1, pg 56-64, Summer 2015, "Factor exposure indexes"; *FTSE Russell Research*, August 2014 http:www.ftse.com/products/downloads/factor-index-construction-methodology-paper.pdf

Exhibit 2. Cumulative Normal Function

Source: FTSE Russell.

Other versions of the tilting function, such as a step function, are equivalent to the characteristic basket selection approach described earlier.

For example, consider the step function that assigns constituents a value of either 0 or 1 based on their factor scores. If stocks are in the top 50% when ranked by factor score, they will be assigned “1”, whereas stocks in the bottom 50% ranked by factor score will be assigned “0” (i.e. excluded from the portfolio). This classic factor portfolio construction technique can be interpreted as a special case of a tilt.

By comparison with this exclusion-based approach, a potential criticism of tilting techniques is that they only provide weak factor exposure. According to this argument, the weights of stocks in the starting universe are only adjusted (upweighted or downweighted), rather than stocks with low factor scores being excluded entirely.

However, this argument can be addressed by noting that the index designer has the ability to apply a factor tilt repeatedly to a set of stock weights. In other words, the index designer has a way to dial up or dial down the relative strength of the tilt via an exponent (or the “power” of the tilt). This compares to the Step function, where the only way to increase or decrease the tilt power would be to vary the percentile level of threshold, e.g. from a top 50% cut off to a top 20% cut off.

The flattening out of extremes in the Cumulative Normal tilting function, as shown in Exhibit 2, now becomes an essential property of the function. If such a flattening did not take place, stocks with large positive Z-scores would rapidly dominate the factor portfolio as the tilt power increased, potentially leading to concentration issues.

Multi-factor indexes

A lively debate has arisen over the appropriate mechanism for obtaining exposure to multiple factors in a single index, whilst maintaining appropriate levels of diversification. Broadly speaking, there are two contrasting approaches to building multi-factor indexes:

Top-down

In a top-down (or "composite") multi-factor index, stock weights are a weighted average of the weights in the component single factor indexes. Below, we examine the properties of a composite of single factor indexes derived using the characteristic basket selection approach, a technique that is widely employed in practice.

Bottom-up

In a bottom-up (or "integrated") multi-factor index, stocks are selected and weighted by taking into account all factor characteristics simultaneously. In this paper we concentrate on the approach favored by FTSE Russell to build multi-factor indexes, called "Tilt-Tilt" or multiple tilting. Under this approach, the index designer applies a tilt to a set of weights that are the result of a previous factor tilt. Mathematically, this is equivalent to multiplying factor scores.

*While diversification is important, the **primary** target of a factor index should be intentional factor exposure.*

Pros and cons of these approaches

The top-down and bottom-up multi-factor index approaches have different merits and drawbacks.

Supporters of the top-down multi-factor index approach suggest it provides the greatest factor exposure consistent with a high degree of diversification. The simplicity of construction also allows for easier performance attribution. Although the averaging of stock weights may result in a weakening of factor exposures (in particular, when factors are negatively correlated), some advocates of the top-down approach argue that high multi-factor exposures can be maintained by averaging higher-exposure single factor portfolios, incorporating more aggressive selection rules. However, such high-exposure single factor portfolios can only be maintained by adversely affecting diversification levels.

Other techniques for solving the problem of weak factor exposure in top-down, composite multi-factor indexes have been put forward. For example, it has been suggested that removing a proportion of stocks with weak multi-factor scores will achieve higher overall factor exposure. However, the removal of stocks that achieve such weak multi-factor scores is equivalent to selecting stocks that score highly, and this approach therefore amounts to, in part, employing a bottom-up approach.

Removing poor multi-factor scoring stocks is trivially equivalent to selecting a proportion of high multi-factor scoring stocks.

Both approaches are "Bottom-Up".

Supporters of the bottom-up approach will argue that attempting to "fix" the top-down approach using elements from a multi-factor scoring and selection approach will not be optimal. It is clear that better results will flow from a position where a bottom-up approach is employed from the start. When designed properly, a factor index that applies the correct tilting function can deliver factor efficiency, that is, improvements in factor exposure per unit of diversification.

Optimization and multi-factor indexes

While not the focus of this piece, optimization has been an important tool in the selection and weighting of stocks since the work of Markowitz.[3] In multi-factor indexes, optimization involves the specification of preferences regarding factor exposures, diversification, risk and constraints and then letting the "black box" do the work of delivering a portfolio that satisfies the desired criteria. If the optimization is designed correctly, and the black box does its job efficiently, the desired outcomes may be reached.

One drawback to optimization is its lack of transparency. The black box knows why it has chosen stocks and in what proportion, but this may not be so clear to the human who allocated it the task. Given this issue, a number of "ad hoc" methods of portfolio construction have also been developed, such as those discussed in this paper

Measuring factor index exposure and concentration

To examine the relationship between exposure and diversification in factor and multi-factor indexes, we must use measures that quantify each. Below, we examine the overall factor exposure of a factor index and its active factor exposure (measured versus a reference index, typically the market capitalization index). For diversification, we use the measure called Effective N.

Effective N is the inverse of the Herfindahl measure of concentration. The Herfindahl index is a commonly used measure of market concentration.

Initially introduced to measure market monopolies and firm size, the concept is also applied to the analysis of index or portfolio diversity.

Factor exposures

Factor exposures are generally measured in one of two ways; via a returns-based regression analysis or via a holdings-based analysis. In this Insights piece, we focus on the holdings-based approach.

The factor exposure (both total and active) of an index is measured in Z-score units. In other words, the index's exposure to a particular factor is the weighted average of the constituent factor Z-scores.

The active factor exposure of an index is then its factor exposure compared to that of a reference index. A common approach is to compare a single or multi-factor index to its corresponding market capitalization-weighted reference index to assess the level of the "active factor bet" that is being taken, and to identify any off-target exposures.

Effective N

The Effective N (i.e. the effective number of stocks) of an index is calculated by squaring the weight of each stock in the index, summing the results and then taking the reciprocal. A larger Effective N means higher levels of diversification (lower levels of concentration).

To illustrate Effective N in practice, consider an equally weighted portfolio (which is the most diverse weighting scheme). Here, the Effective N of the index is equal to the number of stocks in the portfolio. When a portfolio is not equally weighted,

[3] Markowitz, H. "Portfolio Selection", *Journal of Finance*, Vol. 7, No. 1, March 1952, Sharpe, W. "A Simplified Model for Portfolio Analysis", *Management Science*, Vol. 9, p277-293, 163, Black, F. and LItterman, R. "Global Portfolio Optimization", *Financial Analysts Journal*, Vol. 8, No. 5, p28-43, September/October, 1992, Doole, S., Chia, C. Kulkarni, P. and Melas, D. "The MSCI Diversified Multi-Factor Indexes", *MSCI White Paper*, May 2015.

its Effective N is smaller than the number of its constituent stocks. Put another way, the Effective N of a non-equally weighted portfolio represents the number of stocks that *would be* in an equally weighted portfolio *for the same level of diversification*. In the examples below, Effective N will be expressed as a percentage of the total number of stocks in the portfolio or index.

Factor index construction outcomes: Exposure vs. diversification

In this section, we discuss two important outcomes in factor portfolio construction–the aggregate levels of factor exposure and the degree of diversification of the portfolio–and test these properties for theoretical one and two-factor portfolios.[4]

In principle, we should expect aggregate factor exposure and diversification to work in opposite directions. To illustrate this, imagine a portfolio consisting of one stock that has the maximum target factor score. Now imagine an equally weighted portfolio, consisting of all underlying stocks. The one-stock index will exhibit the highest level of exposure to the target factor, but zero diversification. Conversely, the all-stock equally-weighted portfolio would likely display weak exposure (either positive or negative) to the desired factor, but this portfolio would have the highest level of diversification.

We expect aggregate factor exposure and diversification to work in opposite directions.

In the examples shown below, all single factor baskets are equally weighted to give them the maximum possible degree of diversification. All tilted portfolios are correspondingly tilted from equal weight to ensure a like-for-like comparison. Factor values are simulated using a normal distribution. In the multi-factor context, the top-down portfolios studied consist of composites of the single factor baskets. And bottom-up portfolios are constructed using FTSE Russell's multiple tilting ("Tilt-Tilt") approach.

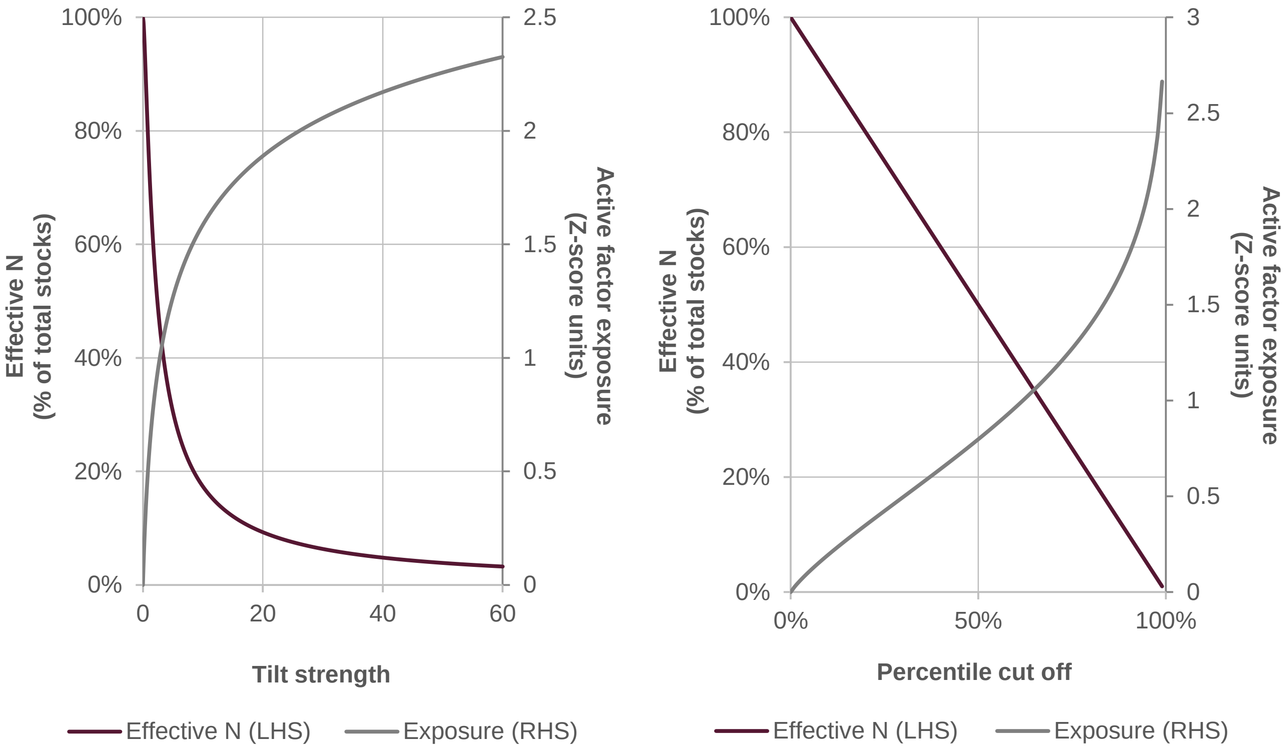
Single factor index

To vary the factor exposure of selection-based portfolios (i.e. characteristic baskets) we select a given percentile of stocks based on their factor values. So, for example, to obtain increased factor exposure we can select stocks above the 80th percentile (i.e., the top 20% of the sample) rather than those in the top half of the sample. For factor tilt portfolios we vary exposure by varying the strength (or power) of the tilt, as indicated in the earlier discussion.

Exhibit 3 shows how the portfolio's factor exposure and diversification vary with the tilt power (left chart) and the percentile at which the cut-off occurs (right chart).

[4] For further analysis on three and N factor scenarios, please see the supporting research, "Factor exposure and portfolio concentration".

Exhibit 3. Factor exposure and diversification of a tilted factor portfolio (left) and selection-based factor portfolio (right)



Source: FTSE Russell.

For the tilt portfolio, we can see that as the strength of the tilt increases (the tilt power goes from 0 – 60), the factor exposure rises sharply and Effective N (diversification) drops sharply. Both measures tend to level out as the tilt power increases.

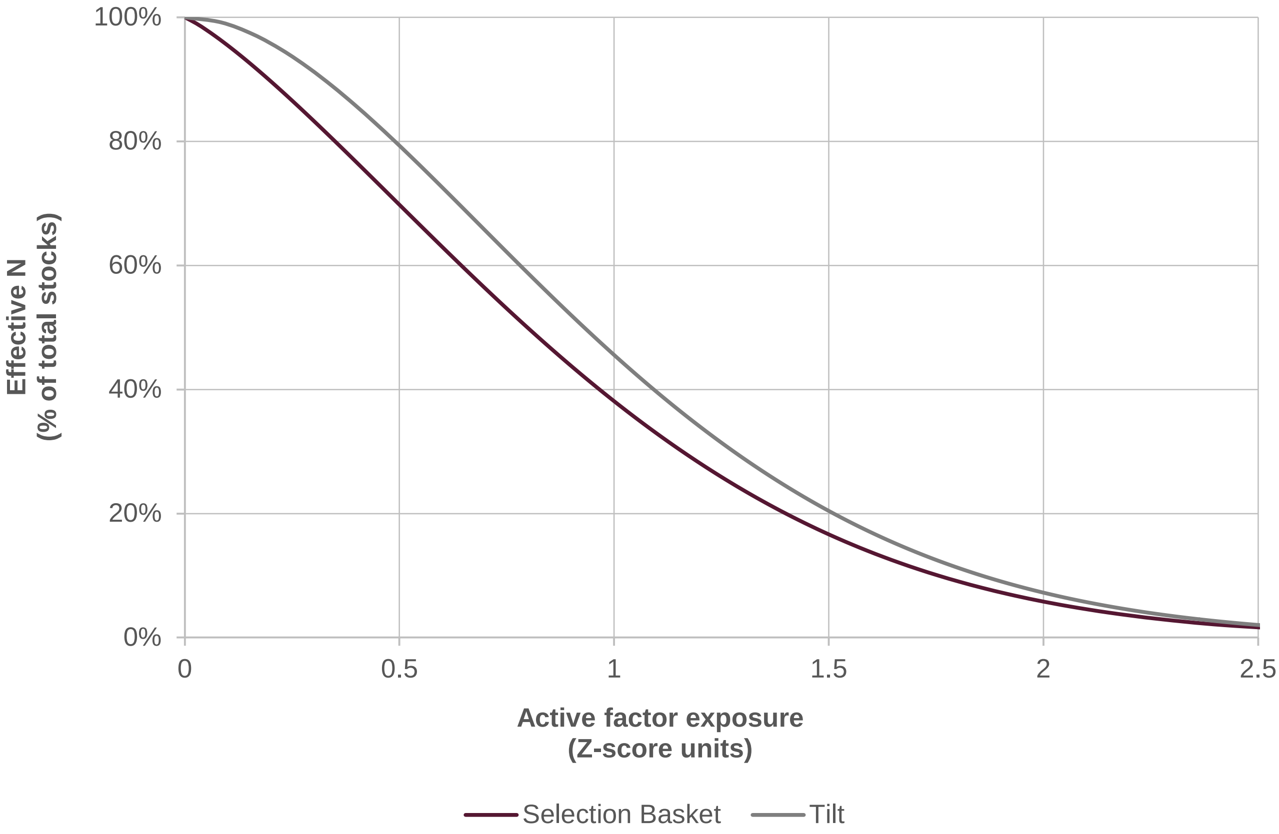
We observe a similar relationship for the selection-based portfolio. As we increase the percentile cut off from 0% towards 100%, a process which is equivalent to narrowing the basket size, the factor exposure increases and the Effective N declines towards 0.

Given the relationship between exposure and diversification for both approaches, an important question is which of the two methods delivers the highest degree of diversification for a given degree of factor exposure.

In other words, which index construction approach is the *most efficient* at achieving factor exposure, while considering a key implementation concern such as diversification? We can answer this question by plotting Effective N as a function of factor exposure for the two index approaches, as shown in Exhibit 4.

Exhibit 4 demonstrates that, for any degree of factor exposure, the tilt portfolio yields a more diverse portfolio than the selection-based approach. And for any level of diversification, the tilt portfolio results in greater levels of exposure to the factor of interest than the selection-based approach.

Exhibit 4. Factor exposure vs Effective N for selection-based and tilted portfolios: single factor index



Source: FTSE Russell.

Multi-factor index

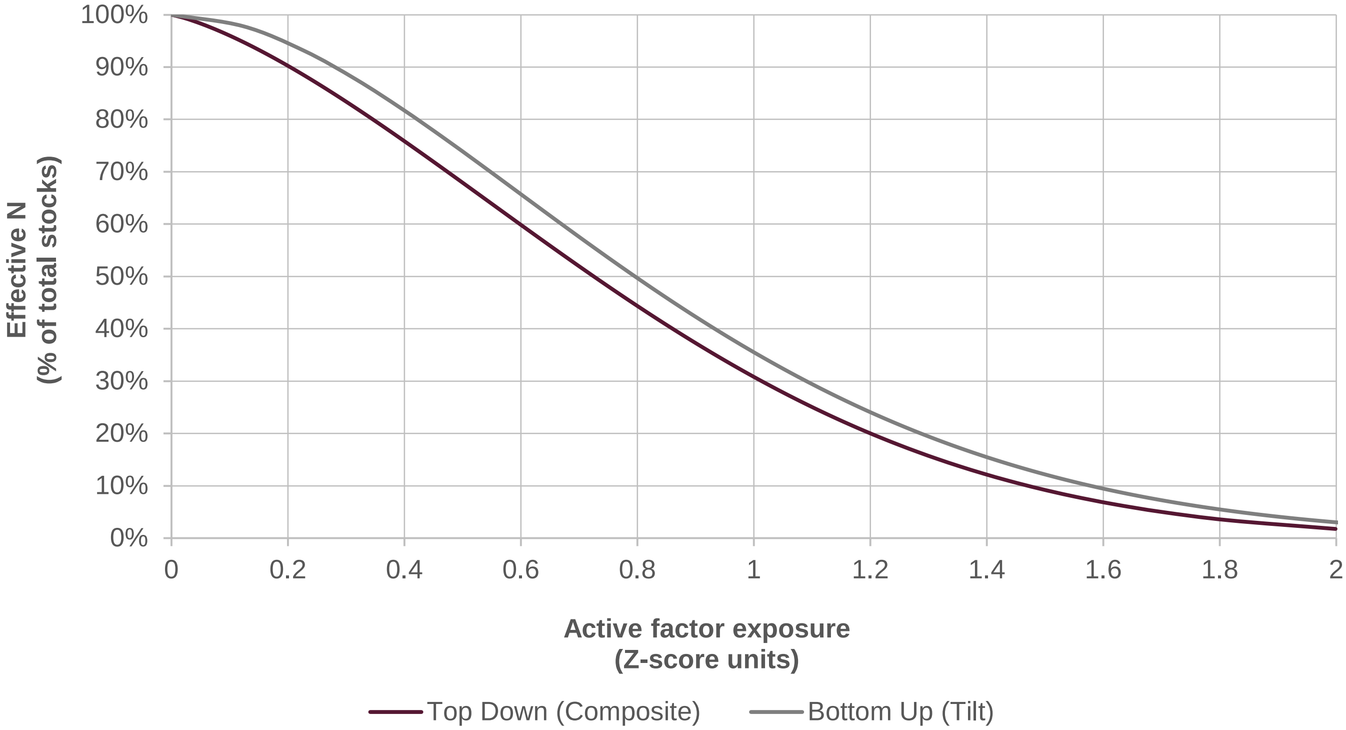
Extending the analysis to a two-factor index allows us to investigate the effect of factor correlation on index construction. We can repeat the previous exercise, but now allow for different degrees of correlation between factors.

It is reasonable to suppose that factor correlation must play a role in determining the two-factor index's factor exposure and diversification, particularly in the case of a negative correlation between the two factors (since we can expect the second factor to cancel out some of the effect of the first factor, and vice versa).

In Exhibits 5, 6 and 7 we calculate the factor exposure and Effective N of two-factor portfolios using different factor correlations: +0.5, 0 and -0.5, respectively.

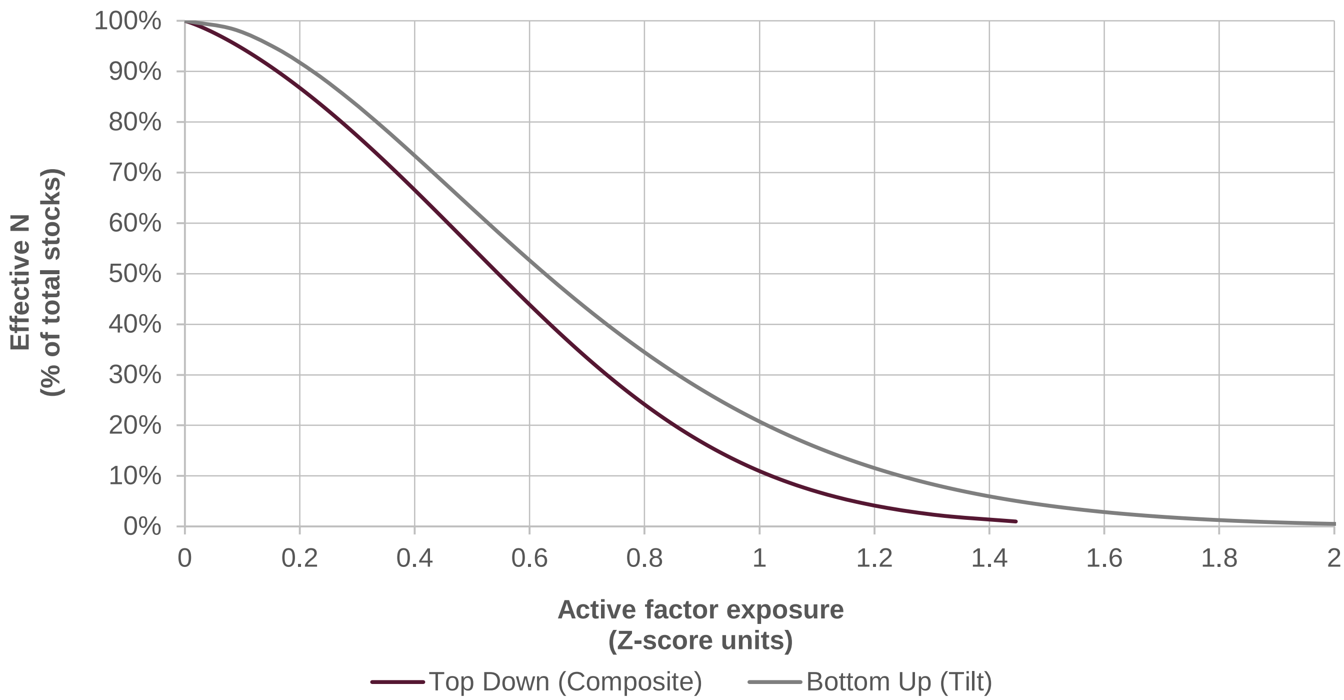
Negatively correlated factors must, in some sense, be in danger of "cancelling one another out".

Exhibit 5: Factor exposure vs Effective N for top-down and bottom-up portfolios: correlation = +0.5



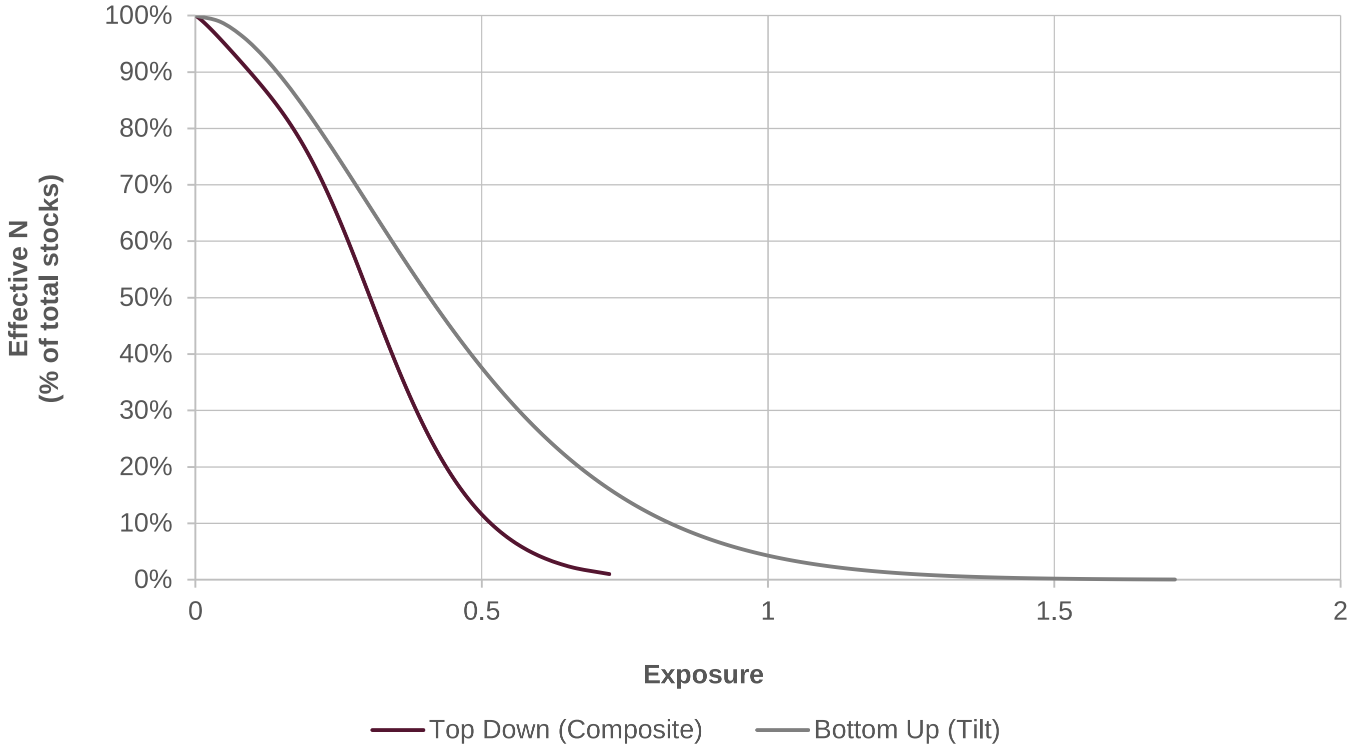
Source: FTSE Russell.

Exhibit 6. Factor exposure vs Effective N for top-down and bottom-up portfolios: correlation = 0.0



Source: FTSE Russell.

Exhibit 7. Factor exposure vs Effective N for top-down and bottom-up portfolios: correlation = -0.5



Source: FTSE Russell.

The results shown in Exhibits 5, 6 and 7 demonstrate that, for all levels of exposure and factor correlation (whether positive, flat, or negative), the bottom-up portfolio yields a more diverse portfolio than the top-down approach. It's also worth noting that as the correlation becomes increasingly negative (that is, moving from +0.5 to -0.5), the advantage of the bottom-up approach becomes increasingly pronounced.

In the supporting technical whitepaper we have calculated the corresponding profiles for the three-factor case and for the N factor-case. In general, as the number of negative correlations increases, the difference between the bottom-up and top-down approaches to combining factors (in terms of the residual factor exposure) becomes more pronounced.

For example, suppose that we require exposure to each factor in a three-factor index of the same magnitude as in each of the three single factor indexes, taken in isolation. In Exhibit 8 we illustrate the levels of diversification produced by the top-down and bottom-up approaches for a three-factor index involving fairly typical pair-wise factor correlations of plus or minus 0.3.

Exhibit 8. Effective N–equivalent single factor levels of exposure for the four correlation combinations

	Effective N	
Correlation combination	**Composite basket**	**Multiple tilt**
+0.3, +0.3, +0.3	54.05%	59.21%
+0.3, +0.3, -0.3	12.06%	42.97%
+0.3, -0.3, -0.3	4.00%	30.61%
+0.3, -0.3, -0.3	0.01%	10.31%

Source: FTSE Russell.

Exhibit 8 shows that the bottom-up portfolio produces better levels of diversification for each correlation combination.

For example, consider a three-factor portfolio targeting Quality, Low Volatility and Value, illustrated by the second line in Exhibit 8 (typically, one finds that Quality and Value are the negatively correlated factor pair).

Here, the difference in diversification levels between the bottom-up and the top-down portfolio are already quite dramatic (42.97% for bottom-up but only 12.06% for top-down).

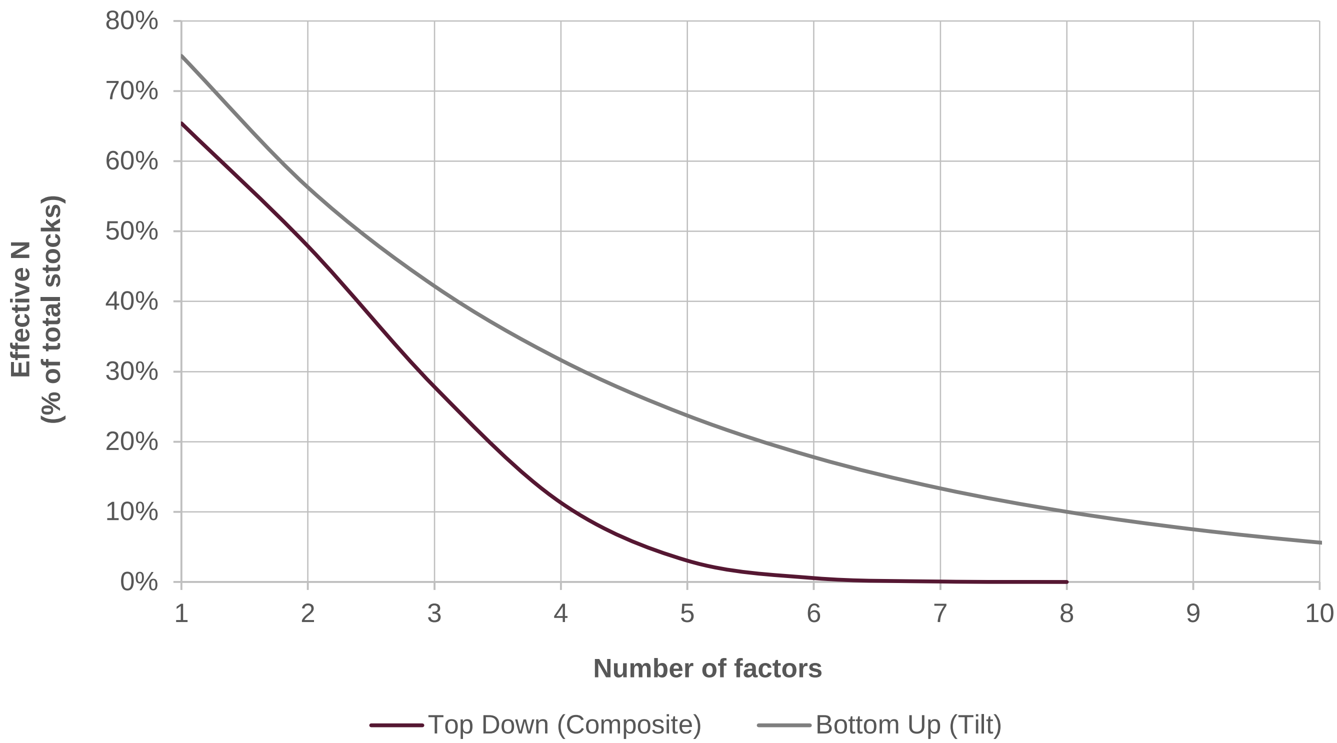
For the case where all three factors are negatively correlated to each other, shown by the last line in Exhibit 8, levels of diversification are essentially zero for the top-down portfolio.

In the case of N-factors, the necessary calculations become complex as they depend on the structure of an N-by-N correlation matrix. However, we can simplify matters considerably by looking at the case where all the factors are uncorrelated. Exhibit 9 shows how, when targeting equivalent levels of factor exposure in the multi-factor index as in each component single factor index, the level of diversification achieved (i.e., the Effective N) varies with the number of factors.

It's evident that the bottom-up portfolio construction approach always results[5] in a greater Effective N than the top-down approach. Furthermore, as the number of factors increases, the percentage difference in diversification between the two approaches becomes greater. Indeed, the Effective N of the top-down approach converges to zero quite rapidly when the number of factors is greater than five.

[5] For a proof, see the paper "Factor Exposure and Portfolio Concentration".

Exhibit 9. Effective N vs number of factors in multi-factor index: top-down and bottom-up approaches



Source: FTSE Russell.

Conclusion

There are a number of approaches to constructing single and multi-factor indexes. Much industry debate centers on which methodology delivers factor exposure in the most efficient way without compromising levels of diversification. In this paper, we have highlighted two popular and contrasting approaches to multi-factor index construction: a top-down combination of selection-based factor portfolios and a bottom-up approach involving multiple tilts towards the factors of interest.

We conducted an analysis of the two approaches, using different hypothetical factor combinations (both in terms of the number of factors involved and the correlations between the factors). We then assessed the index outcomes in terms of factor exposure and diversification.

Our results show that, for a given level of diversification, a tilt approach can deliver stronger exposure to a single factor than a selection-based approach (and, for a given level of exposure, the tilt approach delivers greater diversification). When this analysis is extended to the multi-factor space, bottom-up multiple tilting retains the same diversification advantages over the top-down blending approach, yielding a more efficient factor exposure per unit of diversification.

For more information about our indexes, please visit ftserussell.com.

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